Lecture 4: Scalar-tensor theories and dark energy

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Fields relevant to dark energy

In the standard model of particle physics and cosmology, the most studied fields (which are massive) are spin 0, 1, and 2.

Coupling to gravity

Spin 0 scalar ϕ → Scalar-tensor theories

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V(\phi)$$

(including minimally coupled scalar: quintessence)

Spin 1 vector A_μ → Vector-tensor theories

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - m^2 A_\mu A^\mu$$

Spin2 tensor $g_{\mu\nu}$ → Massive gravity

$$\mathcal{L} = \sqrt{-g}R - m^2\mathcal{U}(g, f)$$

Ostrogradski instability

The theories whose equations of motion contain time derivatives higher than second order are generally prone to an Ostrogradski instability.

Consider the Lagrangian of harmonic oscillator with an additional $\alpha\ddot{q}^2$ term in the Minkowski space-time:

$$L = \frac{1}{2}m\dot{q}^2 - \frac{1}{2}m\omega^2 q^2 + \alpha\ddot{q}^2$$

The Euler-Lagrange equation is

$$\boxed{\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) + \frac{d^2}{dt^2}\left(\frac{\partial L}{\partial \ddot{q}}\right) = 0}$$

For $\alpha \neq 0$, this equation contains derivatives higher than second order.

The Hamiltonian is given by

$$\mathcal{H} = \frac{1}{2}m\dot{q}^2 + \frac{1}{2}m\omega^2 q^2 + \alpha\ddot{q}^2 \underline{- 2\alpha\dot{q}\dddot{q}}$$

This term leads to an instability associated with the Hamiltonian unbounded from below (Ostrogradski instability).

Second-order scalar theories on Minkowski (spin 0 case)

For the scalar field ϕ it is possible to construct the Lagrangian which gives rise to the second-order equations of motion:

$$\boxed{\begin{aligned}
\mathcal{L}_2 &= g_2(\phi, X)\,, \\
\mathcal{L}_3 &= g_3(\phi, X)\Box\phi\,, \\
\mathcal{L}_4 &= g_4(\phi, X)\left[(\Box\phi)^2 - \partial^\mu\partial^\nu\phi\partial_\mu\partial_\nu\phi\right]\,, \\
\mathcal{L}_5 &= g_5(\phi, X)\left[(\Box\phi)^3 - 3(\partial_\mu\partial_\nu\phi)(\partial^\mu\partial^\nu\phi)\Box\phi + 2\partial^\mu\partial_\nu\phi\partial^\nu\partial_\lambda\phi\partial^\lambda\partial_\mu\phi\right].
\end{aligned}}$$

Galileons for $g_{2,3,4,5} \propto X$

where $g_{2,3,4,5}$ are functions of ϕ and $X = -\partial_\mu\phi\partial^\mu\phi/2$.

These Lagrangians can be expressed as

$$\mathcal{L}_N = g_N(\phi, X)\mathcal{E}_{N-1},$$

where

$$\boxed{\mathcal{E}_N = -\mathcal{B}_{(2n+2)}^{\mu_1\cdots\mu_{n+1}\nu_1\cdots\nu_{n+1}}\partial_{\mu_1}\partial_{\nu_1}\phi\cdots\partial_{\mu_n}\partial_{\nu_n}\phi\partial_{\mu_{n+1}}\partial_{\nu_{n+1}}\phi \qquad (\text{with } N = n+2)}$$

with $$\boxed{\mathcal{B}_{(2n)}^{\mu_1\cdots\mu_n\nu_1\cdots\nu_n} = \frac{1}{(4-n)!}\varepsilon^{\mu_1\cdots\mu_n\lambda_1\cdots\lambda_{4-n}}\varepsilon^{\nu_1\cdots\nu_n}{}_{\lambda_1\cdots\lambda_{4-n}}}$$

The anti-symmetric property of the Levi-Civita tensor $\epsilon^{\mu\nu\rho\sigma}$ allows to eliminate derivatives higher than second order from the EOM.

Coupling to gravity

If we try to construct second-order theories in the presence of gravity, the situation is more involved.

Replacing partial derivatives of the second-order Lagrangians $\mathcal{L}_4$, $\mathcal{L}_5$ with covariant derivatives, it gives rise to derivatives higher than second order.

$$\begin{cases} \mathcal{L}_4 = g_4(\phi, X)\left[(\Box\phi)^2 - \nabla^\mu\nabla^\nu\phi\nabla_\mu\nabla_\nu\phi\right] \\ \mathcal{L}_5 = g_5(\phi, X)\left[(\Box\phi)^3 - 3(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi) + 2\nabla^\mu\nabla_\nu\phi\nabla^\nu\nabla_\lambda\phi\nabla^\lambda\nabla_\mu\phi\right] \end{cases}$$

(theories higher than second order)

For $\mathcal{L}_4$ it is possible to eliminate derivatives higher than second order by adding the non-minimal coupling $G_4(\phi, X)R$ with the replacement $g_4(\phi, X) \to G_{4,X} \equiv \partial G_4/\partial X$ (where R is the Ricci scalar).

For $\mathcal{L}_5$ we can also eliminate derivatives higher than second order by adding the gravitational coupling $G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi$ with the replacement $g_5(\phi, X) \to -G_{5,X}/6$ (where $G_{\mu\nu}$ is the Einstein tensor).

$\Rightarrow$ Known as Horndeski theories

Horndeski theories: most general scalar-tensor theories with second-order equations of motion

Horndeski (1974)
Deffayet et al (2011)
Charmousis et al (2011)
Kobayashi et al (2011)

Horndeski action: $S = \int \mathrm{d}^4x\sqrt{-g}\, L$

with the Lagrangian

$$L = G_2(\phi, X) + G_3(\phi, X)\Box\phi + G_4(\phi, X)\, R + G_{4,X}(\phi, X)\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)\right]$$
$$+G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5,X}(\phi, X)\left[(\Box\phi)^3 - 3(\Box\phi)\,(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi) + 2(\nabla^\mu\nabla_\alpha\phi)(\nabla^\alpha\nabla_\beta\phi)(\nabla^\beta\nabla_\mu\phi)\right]$$

This action covers most of the single-field inflation and dark energy models proposed in literature.

- LCDM: $G_2 = -\Lambda, \quad G_3 = 0, \quad G_4 = M_{\rm pl}^2/2, \quad G_5 = 0$
- Quintessence and K-essence: $G_2 = G_2(\phi, X), \quad G_3 = 0, \quad G_4 = M_{\rm pl}^2/2, \quad G_5 = 0$
- f(R) gravity and scalar-tensor gravity: $G_4 = F(\phi), \quad G_3 = 0, \quad G_5 = 0$
- Galileons: $G_2 = -c_2X, \quad G_3 = c_3X/M^3, \quad G_4 = M_{\rm pl}^2/2 - c_4X^2/M^6, \quad G_5 = 3c_5X^2/M^9$
- Gauss-Bonnet coupling $\xi(\phi)\mathcal{G}$:

$$G_2 = 8\xi^{(4)}(\phi)X^2(3 - \ln X), \quad G_3 = 4\xi^{(3)}(\phi)X(7 - 3\ln X)$$
$$G_4 = 4\xi^{(2)}(\phi)X(2 - \ln X), \quad G_5 = -4\xi^{(1)}(\phi)\ln X$$

Horndeski's paper in 1973 (age 25)

When Horndeski was the PhD student of David Lovelock, he wrote this valuable paper.

International Journal of Theoretical Physics, Vol. 10, No. 6 (1974), pp. 363–384

Second-Order Scalar-Tensor Field Equations in a Four-Dimensional Space

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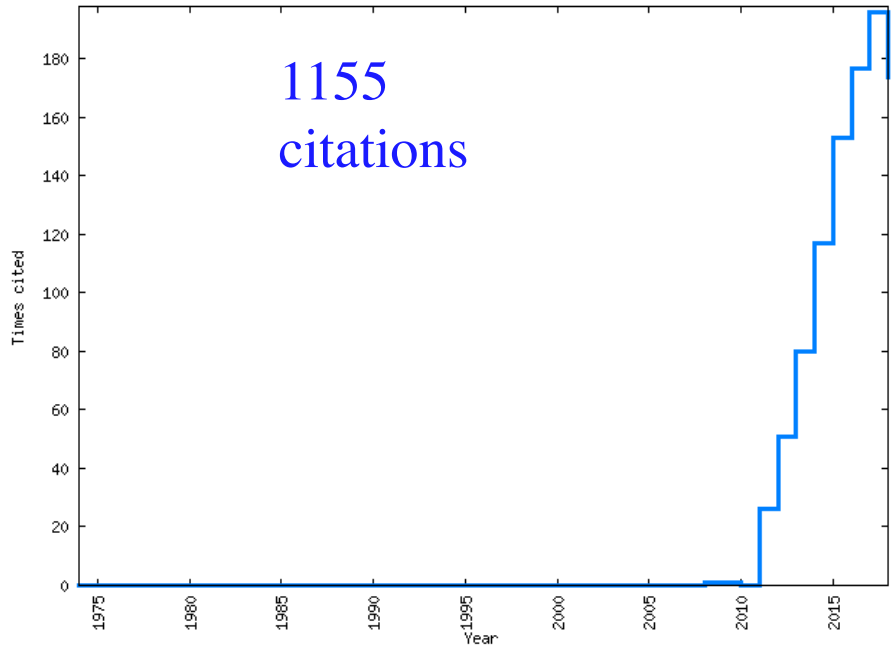
Received: 10 *July* 1973

Abstract

Lagrange scalar densities which are concomitants of a pseudo-Riemannian metric-tenso a scalar field and their derivatives of arbitrary order are considered. The most general second-order Euler–Lagrange tensors derivable from such a Lagrangian in a four-dimensional space are constructed, and it is shown that these Euler–Lagrange tensors may be obtained from a Lagrangian which is at most of second order in the derivatives of the field functions.



After 1981, Horndeski became an artist.

Horndeski's paper in 2016 (age 68)

arXiv:1608.03212

Lagrange Multipliers

and

Third Order

Scalar-Tensor Field Theories

by

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September 10, 2016

Now, Horndeski wants to construct scalar-tensor theories with third-order equations of motion.

Without taking care of the Ostrogradski instability…

Friedmann equations on the flat FLRW background

$$S = \int \mathrm{d}^4x\sqrt{-g}\,[G_2(\phi, X) + G_3(\phi, X)\Box\phi + L_4 + L_5] + S_m + S_r$$

(Horndeski's action)

Non-relativistic matter

Radiation

The background equations of motion are

$$3M_{\rm pl}^2 H^2 = \rho_{\rm DE} + \rho_m + \rho_r$$

$$-2M_{\rm pl}^2 \dot{H} = \rho_{\rm DE} + P_{\rm DE} + \rho_m + 4\rho_r/3$$

$\rho_{\rm DE}$ and $P_{\rm DE}$ are the density and pressure of the "dark" component.

$$\rho_{\rm DE} = 3H^2\left(M_{\rm pl}^2 - 2G_4\right) - G_2 + \dot{\phi}^2 G_{2,X} - \dot{\phi}^2\left(3H\dot{\phi}G_{3,X} - G_{3,\phi}\right) - 6H\dot{\phi}\left(G_{4,\phi} + \dot{\phi}^2 G_{4,X\phi}\right.$$
$$\left. -2H\dot{\phi}G_{4,X} - H\dot{\phi}^3 G_{4,XX}\right) - H^2\dot{\phi}^2\left(9G_{5,\phi} + 3\dot{\phi}^2 G_{5,X\phi} - 5H\dot{\phi}G_{5,X} - H\dot{\phi}^3 G_{5,XX}\right)$$

See arXiv: 1809.08735 for details.

The equation of state of dark energy is given by

$$w = P_{\rm DE}/\rho_{\rm DE}$$

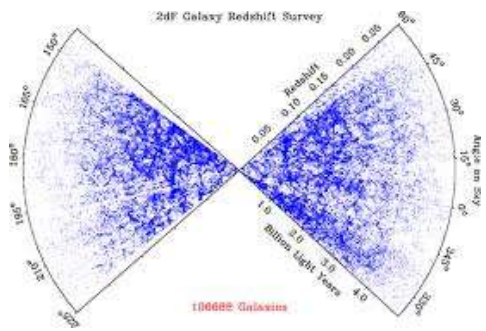
The evolution of w is different depending on dark energy models.

Different models can be distinguished from SNIa, CMB, and BAO data.

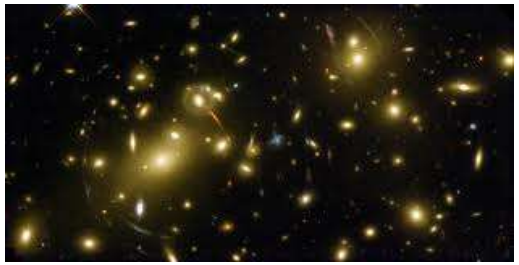
Discrimination of models from scalar perturbations

In order to place constraints on dark energy models from the observations of large-scale structure, weak lensing, CMB (ISW effect) etc, we need to study the evolution of scalar density perturbations.

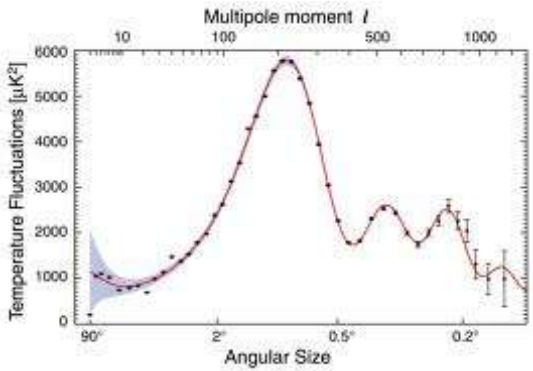
LSS



Weak lensing



CMB



Perturbed metric: $ds^2 = -(1+2\Psi)dt^2 + a^2(t)(1+2\Phi)\delta_{ij}dx^i dx^j$

Non-relativistic matter: $\rho_m = \rho_m(t) + \delta\rho_m(t, \mathbf{x})$

with the four velocity $u^\mu = (1-\Psi, \nabla^i v)$

v is the rotational-free velocity potential.

Density perturbations in the Horndeski's theory

$\delta \equiv \delta\rho_m/\rho_m$ and $\theta \equiv \nabla^2 v$ obey (in Fourier space with the comoving wavenumber k)

$$\begin{cases} \dot{\delta} = -\theta/a - 3\dot{\Phi} \\ \dot{\theta} = -H\theta + (k^2/a)\Psi \end{cases}$$

The growth rate of matter perturbations is related with the peculiar velocity.

We introduce the gauge-invariant density contrast: $\delta_m \equiv \delta + \frac{3aH}{k^2}\theta$

$$\ddot{\delta}_m + 2H\dot{\delta}_m + \underline{\frac{k^2}{a^2}\Psi} = 3\left(\ddot{I} + 2H\dot{I}\right) \qquad \text{where} \quad I \equiv (aH/k^2)\theta - \Phi$$

Source term

The gravitational potential Ψ is generally related to $\delta\rho_m$, as

$$\boxed{\frac{k^2}{a^2}\Psi = -4\pi G\underline{\mu}\delta\rho_m}$$

In GR $\mu = 1$, but in Horndeski theories μ is generally different from 1.

The matter density contrast δ_m obeys

$$\boxed{\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi\underline{\mu}G\rho_m\delta_m \simeq 0}$$

Different models can be distinguished from different evolution of δ_m.

Lensing gravitational potential

To characterize the bending of light rays in CMB and weak lensing observations, we define the lensing potential

$$\psi_{\rm eff} = \Phi - \Psi$$

This is related to the matter perturbation, as

$$\boxed{\frac{k^2}{a^2}\psi_{\rm eff} = 8\pi G \Sigma \delta\rho_m}$$

In GR $\Sigma = 1$, but in Horndeski theories Σ is generally different from 1.

What are the values of μ and Σ in Horndeski theories?

This was generally derived by De Felice, Kobayashi, S.T. (2011), see also 1809.08735.

Quasi-static approximation

See 1809.08735.

Under the quasi-static approximation under which the dominant contributions to the perturbation equations are those containing k^2 and $\delta\rho_m$, we obtain

$$\mu = \frac{c_t^2}{8\pi G q_t}\left(1 + \frac{q_t \Delta_1^2}{c_t^2 \Delta_2}\right)$$

$$\begin{cases} \Delta_1 \equiv c_t^2(1+\alpha_{\rm B}) - 1 - \alpha_{\rm M} \\ \Delta_2 \equiv \dfrac{\dot{\phi}^2 q_s c_s^2}{4H^2 q_t}\left(1 + \dfrac{2a^2 M_\phi^2 q_t}{c_s^2 k^2 q_s}\right) \end{cases}$$

Arising from tensor perturbations

Scalar-matter interactions

Under the absence of ghosts and Laplacian instabilities of scalar and tensor perturbations, the gravitational interaction felt by matter is always enhanced.

$$\mu > \frac{c_t^2}{8\pi G q_t}$$

The expression of Σ is

$$\Sigma = \frac{1+c_t^2}{16\pi G q_t}\left[1 + \frac{q_t(\alpha_{\rm B} + \Delta_1)\Delta_1}{(1+c_t^2)\Delta_2}\right]$$

This can be larger or smaller than 1.

Gravitational-wave propagation in Horndeski theories

$$ds^2 = -dt^2 + a^2(t)(\delta_{ij} + \underline{h_{ij}})dx^i dx^j$$

Tensor perturbations obeying $h^i_i = 0, \quad \partial^i h_{ij} = 0$

We can choose the nonvanishing components:

$$h_{11} = h_1(t,z)\,, \qquad h_{22} = -h_1(t,z)\,, \qquad h_{12} = h_{21} = h_2(t,z)$$

The second-order action of Horndeski theories expanded up to second order is

$$\boxed{\mathcal{S}_t^{(2)} = \int dt d^3x \sum_{i=1}^{2} \frac{a^3}{4} q_t \left[\dot{h}_i^2 - \frac{c_t^2}{a^2}(\partial h_i)^2 \right]}$$

where

$$\begin{cases} q_t = 2G_4 - 2\dot{\phi}^2 G_{4,X} + \dot{\phi}^2 G_{5,\phi} - H\dot{\phi}^3 G_{5,X} & > 0 \text{ to avoid ghosts} \\ c_t^2 = \dfrac{1}{q_t}\left(2G_4 - \dot{\phi}^2 G_{5,\phi} - \dot{\phi}^2 \ddot{\phi} G_{5,X}\right) & > 0 \text{ to avoid Laplacian instability} \end{cases}$$

Here, c_t correspond to the propagation speed of gravitational waves.

Gravitational wave speed constraints on dark energy

The gravitational-wave event GW170817 constrained the speed of gravitational waves to be very close to that of light.

$$-3 \times 10^{-15} \leq c_t/c - 1 \leq 7 \times 10^{-16}$$

while $$c_t^2 = \frac{2G_4 - \dot{\phi}^2 G_{5,\phi} - \dot{\phi}^2 \ddot{\phi} G_{5,X}}{2G_4 - 2\dot{\phi}^2 G_{4,X} + \dot{\phi}^2 G_{5,\phi} - H\dot{\phi}^3 G_{5,X}}$$ in Horndeski theories

$$\begin{aligned} L &= G_2(\phi, X) + G_3(\phi, X)\Box\phi + G_4(\phi, X)R - \cancel{2G_{4,X}(\phi, X)\left[(\Box\phi)^2 - \phi^{;\mu\nu}\phi_{;\mu\nu}\right]} \\ &\quad \cancel{+G_5(\phi, X)G_{\mu\nu}\phi^{;\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X)[(\Box\phi)^3 - 3(\Box\phi)\phi_{;\mu\nu}\phi^{;\mu\nu} + 2\phi_{;\mu\nu}\phi^{;\mu\sigma}\phi^{;\nu}{}_{;\sigma}]} \end{aligned}$$

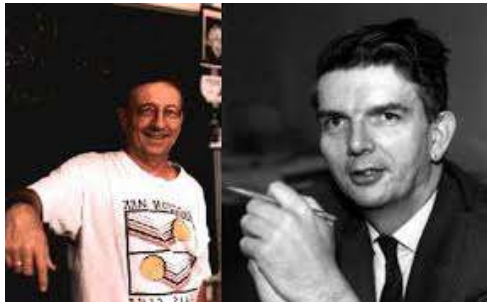
If we strictly demand
$c_t = c$

$$L = \underbrace{G_2(\phi, X)}_{\text{Quintessence, K-essence}} + \underbrace{G_3(\phi, X)\Box\phi}_{\text{Cubic Galileons}} + \underbrace{G_4(\phi)R}_{\text{Brans-Dicke theory, f(R) gravity}}$$

$G_4 = M_{\rm pl}^2/2$ in GR

In the following, let's focus on this reduced Lagrangian.

Brans-Dicke (BD) theory (1961)



Lagrangian: $$L = \frac{M_{\rm pl}^2}{2} F(\phi) R + \left(1 - 6Q^2\right) F(\phi) X$$

where $$F(\phi) = e^{-2Q\phi/M_{\rm pl}}$$

The constant Q characterizes the coupling between the scalar field ϕ and matter in the Einstein frame. It is related to the BD parameter $\omega_{\rm BD}$, as

$$Q^2 = \frac{1}{2(3 + 2\omega_{\rm BD})}$$

GR is recovered for $\omega_{\rm BD} \to \infty$ i.e., $Q \to 0$

The coupling Q mediates fifth forces. The solar system experiment gives

$$\omega_{\rm BD} > 40000 \quad \Rightarrow \quad |Q| < 2.4 \times 10^{-3}$$

For $|Q| > 2.4 \times 10^{-3}$, we need some screening mechanism of fifth forces. Two examples are

(1) Chameleon mechanism: Based on the scalar potential $V(\phi)$

e.g., $f(R)$ gravity

(2) Vainshtein mechanism: Based on the derivative coupling $G_3(X)\Box\phi$

e.g., Cubic Galileon $X\Box\phi$

f(R) gravity (chameleon mechanism)

Strarobinsky (1980), Capozziello (2003), Carroll et al. (2003)

The $f(R)$ gravity is equivalent to BD theories with $Q = -1/\sqrt{6}$ in the presence of a scalar potential:

$$V = \frac{M_{\rm pl}^2}{2}\left(R\frac{\partial f}{\partial R} - f\right)$$

with the scalar degree of freedom (scalaron): $\phi = \sqrt{\frac{3}{2}} M_{\rm pl} \ln \frac{\partial f}{\partial R}$

As long as the form of $f(R)$ is designed to have a large mass in regions of high density, the chameleon mechanism is at work.

Example: $$f(R) = R - \lambda R_0 \frac{(R/R_0)^{2n}}{(R/R_0)^{2n} + 1}$$ (Hu and Sawicki, 2007)

In the high-density region ($R \gg R_0$), the scalaron mass squared grows as

$$M_\phi^2 = \frac{d^2 V}{d\phi^2} \propto R^{2(n+1)} \gg H^2$$

The field is very heavy, so the propagation of fifth forces is suppressed.

f(R) dark energy

Amendola et al, Hu and Sawicki, Starobinsky, Appleby and Battye, ST (2007),…
(more than 1000 papers)

The models are constructed to recover the ΛCDM behavior in the past.

$$f(R) = R - \lambda R_0 \frac{(R/R_0)^{2n}}{(R/R_0)^{2n}+1} \quad \Rightarrow \quad f(R) = R - \lambda R_0 \quad \text{for} \quad R \gg R_0$$

After R decreases to the order of R_0, the model deviates from the ΛCDM.

Deviation parameter from the ΛCDM:

$$B = \frac{R f_{,RR}}{f_{,R}} \frac{H}{\dot{H}} \frac{\dot{R}}{R} \quad \Rightarrow \quad B < 1.1 \times 10^{-3} \quad \text{today}$$

Lombriser et al (2012)

To avoid the large enhancement of perturbations at the late cosmological epoch ($G_{\rm eff} = 4G/3$).

The variation of w at low redshifts is also limited:

$$|w+1| < \mathcal{O}(0.01) \quad \Rightarrow$$

Battye et al (2018)

Indistinguishable from the ΛCDM in current observations.

Cubic Galileons (Vainshtein mechanism)

$$L = X - m^3\phi + \frac{\beta_3}{M^3} X\Box\phi + \frac{M_{\rm pl}^2}{2} e^{-2Q\phi/M_{\rm pl}} R$$

where

$M = (M_{\rm pl} H_0^2)^{1/3} \simeq 10^{-22}$ GeV

Around a local source with the Schwarzschild radius r_g, the scalar-matter coupling is suppressed within the Vainshtein radius:

$$r_V = \left(|\beta_3 Q| r_g H_0^{-2}\right)^{1/3}$$

See arXiv:1111.5090, arXiv:1111.6749

As long as

$$|\beta_3 Q| \gg 10^{-17}$$

the firth force is suppressed inside the solar system.

Line element: $\mathrm{d}s^2 = -\left(1+2\Psi\right)\mathrm{d}t^2 + \left(1+2\Phi\right) a^2(t)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$

Inside the Vainshtein radius,

$$-\Psi \simeq \Phi \simeq \frac{G_{\rm N}\mathcal{M}}{r}\left[1 + \underline{\mathcal{O}(1)Q^2\left(\frac{r}{r_V}\right)^{3/2}}\right]$$

where $G_{\rm N} = \dfrac{1}{8\pi M_{\rm pl}^2 \underline{F(\phi)}}$

suppressed for $r \ll r_V$

Inheriting the time dependence of dark energy scalar field

Lunar Laser Ranging (LLR) bound

Hofmann and Muller, 2018

The recent LLR bound on the time variation of $G_{\rm N}$ today is

$$\frac{\dot{G}_{\rm N}}{G_{\rm N}} = (7.1 \pm 7.6) \times 10^{-14}\ {\rm yr}^{-1}$$

Tighter than the previous one by one order of magnitude

Nonminimally coupled theories: $G_{\rm N} = \dfrac{1}{8\pi M_{\rm pl}^2 F(\phi)}$

$$\Rightarrow \quad \frac{\dot{G}_{\rm N}}{G_{\rm N}} = -\frac{\dot{F}}{F}$$

The Vainshtein mechanism does not screen the time variation of $G_{\rm N}$.

Deffayet et al, 1107.1569 (2011)

Defining $\alpha_{\rm M} \equiv \dfrac{\dot{F}}{HF} = -\dfrac{2Q\dot{\phi}}{HM_{\rm pl}}$, today's LLR bound translates to

$$-2.05 \times 10^{-3} \left(\frac{0.7}{h}\right) \leq \alpha_{\rm M}(t_0) \leq 0.07 \times 10^{-3} \left(\frac{0.7}{h}\right)$$

where $H_0 = 100\ h\ {\rm km\ s^{-1}\ Mpc}$ is today's Hubble expansion rate.

This strongly limits the time variation of the dark energy field ϕ nonminimally coupled to gravity.

Constraints on the nonminimal coupling of cubic Galileons

$$L = X - m^3\phi + \frac{\beta_3}{M^3} X\Box\phi + \frac{M_{\rm pl}^2}{2} e^{-2Q\phi/M_{\rm pl}} R$$

ST, arXiv:1903.07092

- For $|\beta_3| \ll 1$, the matter era is replaced by the ϕMDE in which the field kinetic energy dominates the scalar field dynamics.

$$\alpha_{\rm M} \simeq 4Q^2 < 0.07 \times 10^{-3} \quad \Rightarrow \quad |Q| < 4.2 \times 10^{-3}$$

On using today's bound $\alpha_{\rm M}(t_0) < 0.07 \times 10^{-3}$, the bound is even tighter:

$$|Q| < 3.4 \times 10^{-3} \quad \Rightarrow$$

Almost close to the bound $|Q| < 2.4 \times 10^{-3}$ derived without the Vainshtein screening.

- For $|\beta_3| \gg 1$, the ϕMDE is not present and the field kinetic energy is strongly suppressed due to the cosmological Vainshtein screening.

$\Rightarrow$ $|Q|$ is not strongly constrained, but the time variation of ϕ is very tiny.

$\Rightarrow$ The nonminimal coupling $F(\phi) = e^{-2Q\phi/M_{\rm pl}}$ is very close to 1.

Modified gravitational wave (GW) propagation

Amendola et al (2018), Maggiore et al (2018),…

In nonminimally coupled theories, tensor perturbations obey

$$\ddot{h}_{ij} + H\left(3 + \underline{\alpha_{\rm M}}\right)\dot{h}_{ij} + \frac{k^2}{a^2}h_{ij} = 0$$

Modified from GR

The GW and luminosity distances are different:

$$d_{\rm GW}(z) = \frac{d_L(z)}{\sqrt{F(z)}}$$

$$\frac{d_{\rm GW}(z)}{d_L(z)} - 1 \lesssim \frac{\alpha_{\rm max}}{2}\ln(z+1) \lesssim 3.5\times 10^{-5}\ln\left(1+z\right)$$

LLR bound: $\alpha_{\rm max} = 0.07\times 10^{-3}$

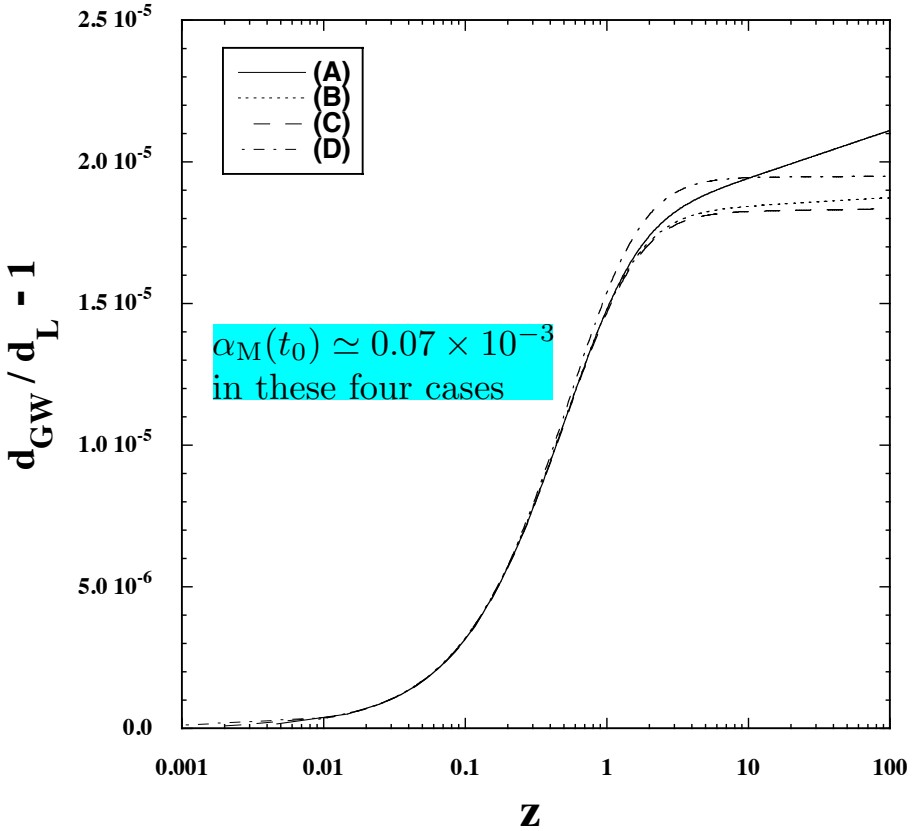
The upper bound is determined by the value of $\alpha_{\rm M}$ around $z \lesssim 1$, so that

$$\frac{d_{\rm GW}(z)}{d_L(z)} - 1 \lesssim \mathcal{O}(10^{-5}) \qquad \text{for} \quad 0 < z < 100$$

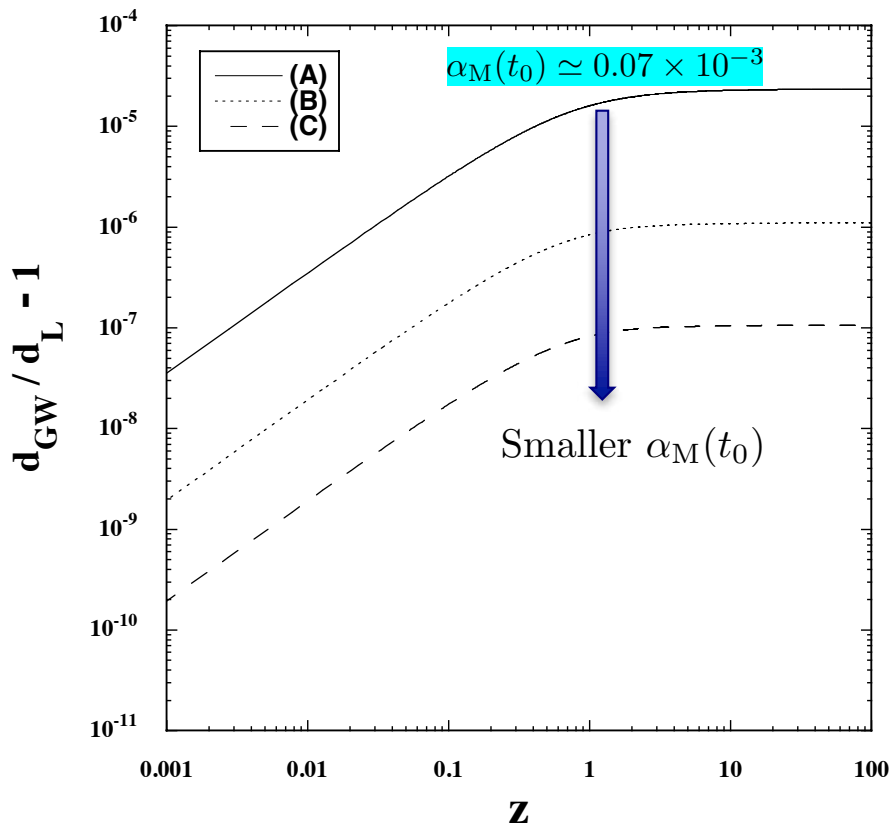
Model: $L = X - m^3\phi + \frac{\beta_3}{M^3} X\Box\phi + \frac{M_{\rm pl}^2}{2} e^{-2Q\phi/M_{\rm pl}} R$

ST, arXiv:1903.07092

$|\beta_3| \ll 1$



$|\beta_3| \gg 1$



In both cases, $\frac{d_{\rm GW}(z)}{d_L(z)} - 1 \lesssim \mathcal{O}(10^{-5})$ → Difficult to distinguish two distances observationally under the LLR bound

Minimally coupled cubic Horndeski theories

If we do not observe any signatures of nonminimal couplings, the left-over Horndeski Lagrangian is

$$L = G_2(\phi, X) + G_3(\phi, X)\Box\phi + \frac{M_{\rm pl}^2}{2}R$$

There are three possibilities (in the presence of cubic Lagrangian):

(A) Galileons without a potential: $L = X + \frac{\beta_3}{M^3}X\Box\phi + \frac{M_{\rm pl}^2}{2}R$ ⇨ Ruled out

There exists the self-accelerating solution with $\dot{\phi} = \text{constant}$.

(B) Galileons with a potential: $L = X - V(\phi) + \frac{\beta_3}{M^3}X\Box\phi + \frac{M_{\rm pl}^2}{2}R$

Galileon has a linear potential $V(\phi) = m^3\phi$ driving cosmic acceleration.

(C) Galileons with k-essence: $L = G_2(X) + \frac{\beta_3}{M^3}X\Box\phi + \frac{M_{\rm pl}^2}{2}R$

For example, the ghost condensate $G_2(X) = -X + c_2X^2$ leads to the dark energy dynamics different from case (A).

(A) Galileons without a potential

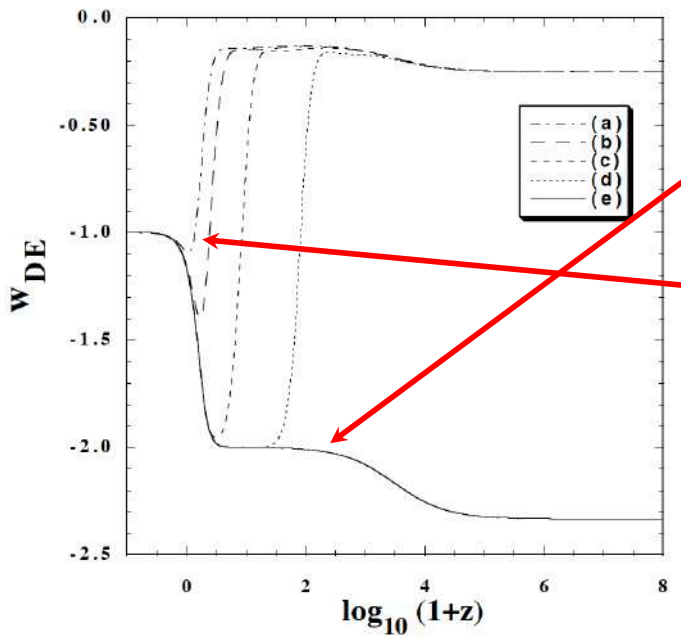
Lagrangian: $$L = X + \frac{\beta_3}{M^3} X \Box \phi + \frac{M_{\rm pl}^2}{2} R$$

There is a tracker solution along which $w = -2$ in the matter era (finally approaching $w = -1$).

De Felice and ST (2010)

Disfavored from the CMB+BAO+SNe data

Nesseris, De Felice, ST (2010)



Tracker solution
(tension with the background data)

Late-time tracking solution
(fine at the background level, but it is still in tension with the cosmic growth history)

The cosmic growth history of Galileons is also in tension with the observational data of redshift-space distortions, weak lensing, and ISW-galaxy cross-correlations.

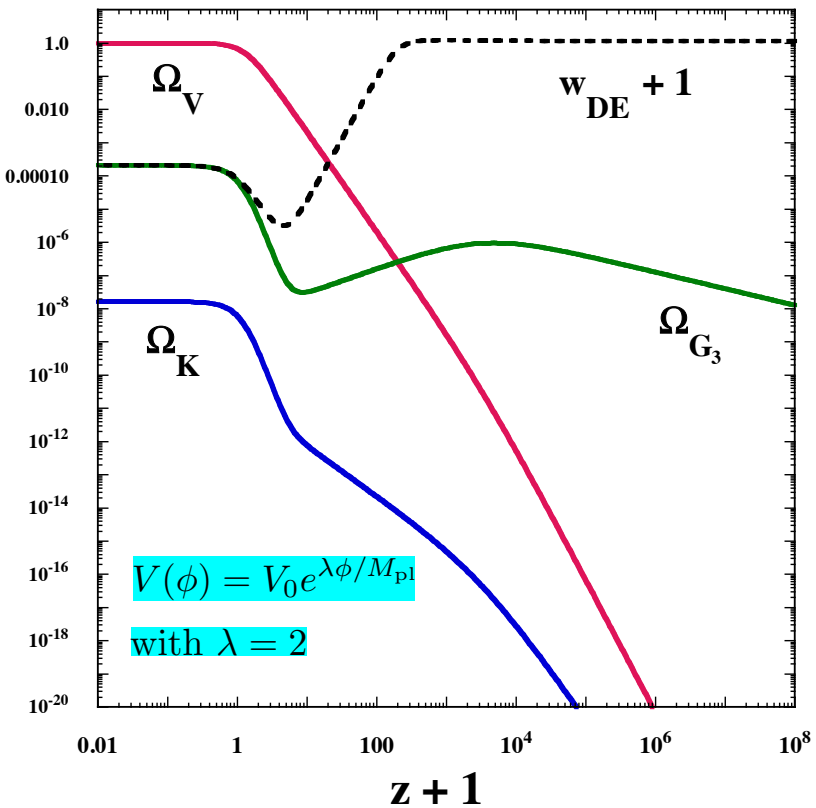
Renk et al (2016)

(B) Galileons with a potential

$$L = X - V(\phi) + \frac{\beta_3}{M^3} X \Box\phi + \frac{M_{\rm pl}^2}{2} R$$

Provided the potential $V(\phi)$ of a light scalar dominates over the Galilen term at late times, the model is observationally allowed.

Bound on today's Galileon density parameter: $\Omega_{G_3}(t_0) < 0.2$



For $\beta_3 > 1$, the Galileon term can suppress the field kinetic energy such that $\Omega_K \ll \Omega_{G_3} \ll \Omega_V = \mathcal{O}(1)$ today.

Even for $\lambda \equiv M_{\rm pl} V_{,\phi}/V > 1$, the dark energy equation of state quickly approaches -1 after the dominance of Ω_V (with $w_{\rm DE} > -1$).

It deserves for further study to find the allowed parameters space of

β_3 and λ

(C) Galileon ghost condensate (GGC)

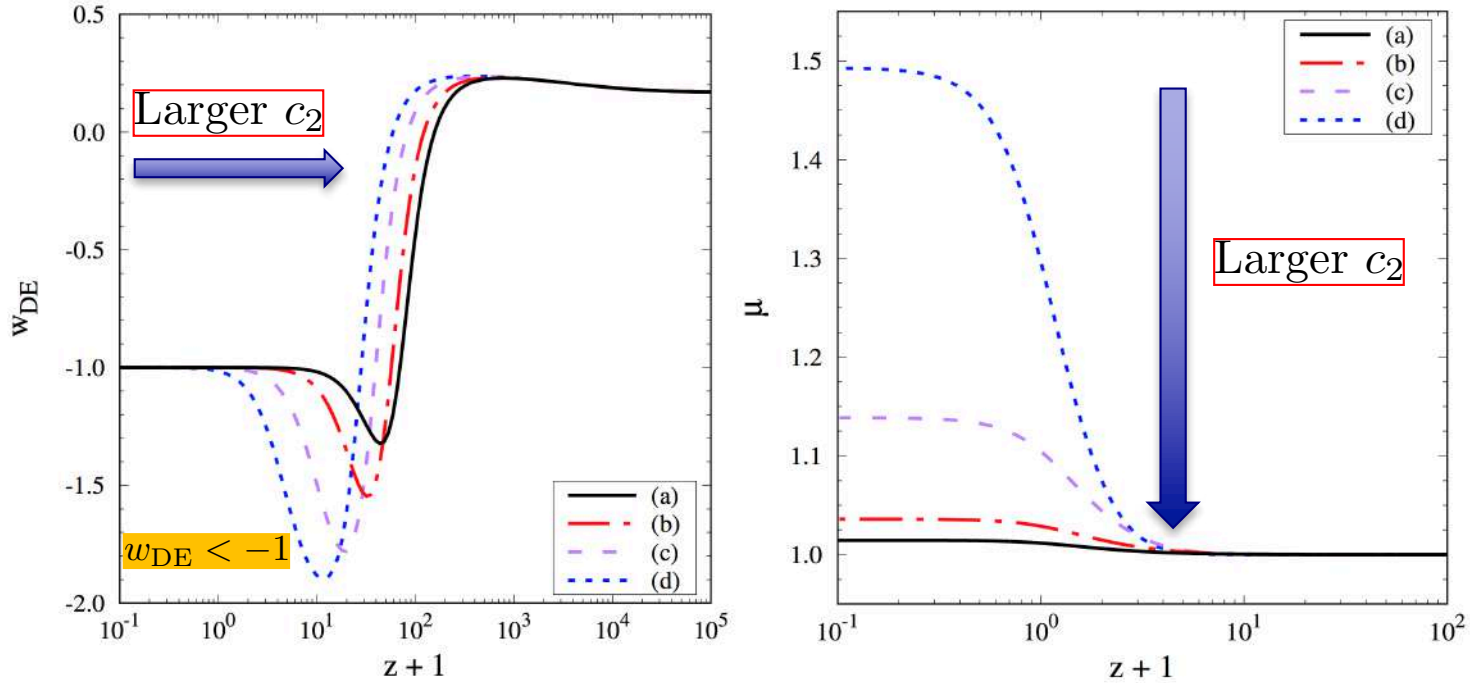
Kase and ST (2018)

$$L = X + c_2 X^2 + \frac{\beta_3}{M^3}\Box\phi + \frac{M_{\rm pl}^2}{2}R$$

Ghost condensate + Galileon

This term prevents the approach to tracker solutions ($w_{\rm DE} = -2$).

Moreover, the growth perturbations can be close to that in GR ($\mu = \Sigma = 1$).



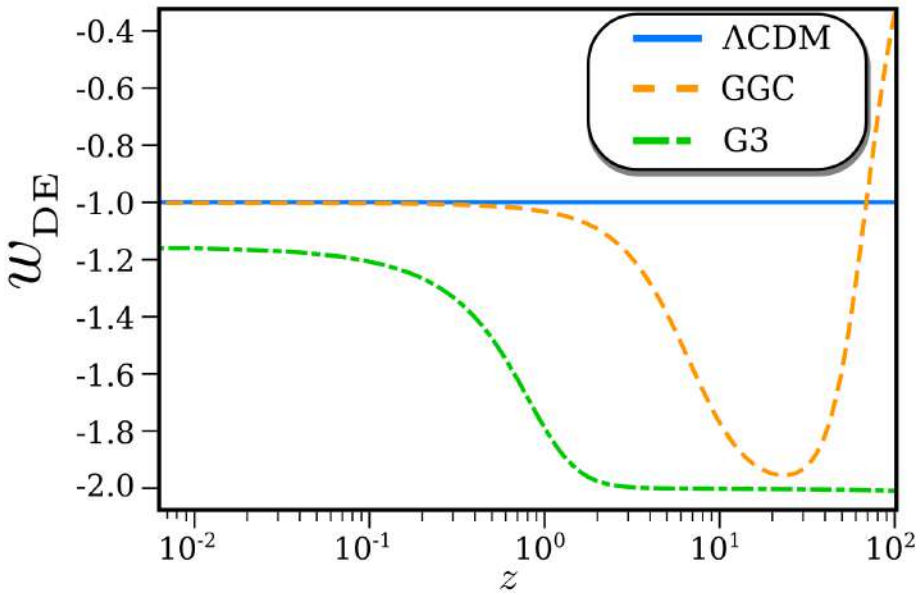
There should be an observational lower bound on c_2 (under consideration).

GGC is observationally favored over LCDM

Peirone et al, arXiv:1905.05166

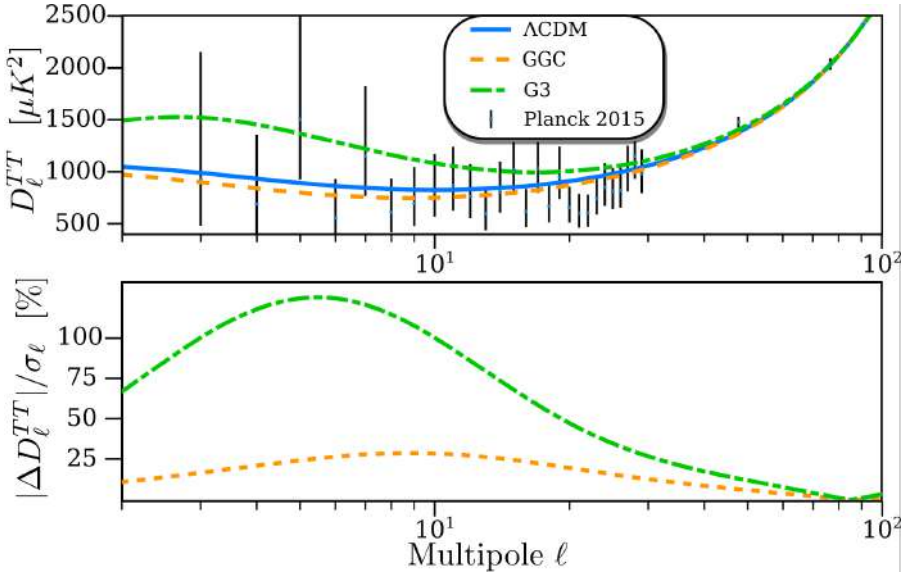
According to the joint analysis of SN Ia, CMB, BAO, and RSD data, GGC is statistically favored over the LCDM.

Background



At the background, $w_{\rm DE}$ is in the range $-2 < w_{\rm DE} < -1$.

CMB spectrum (large scales)



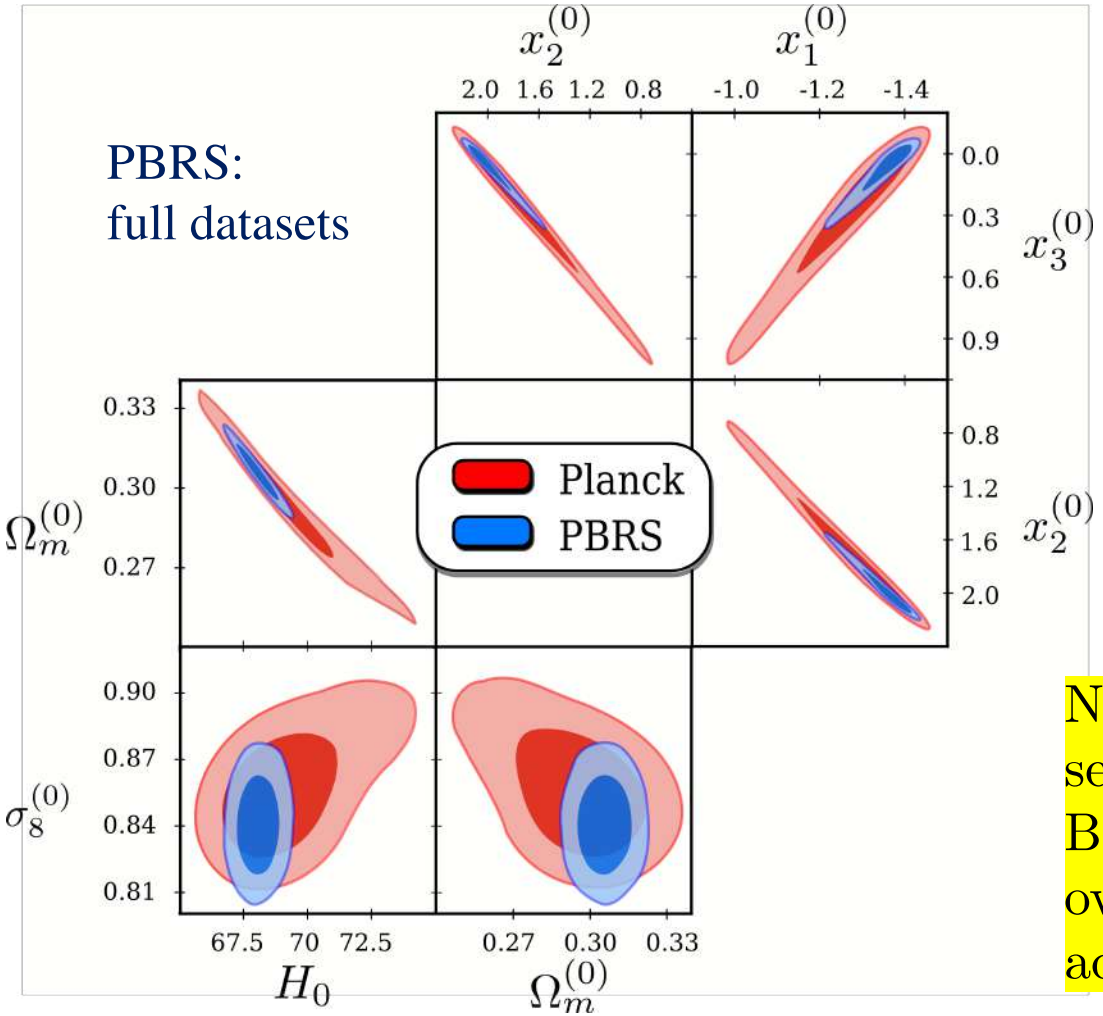
The large-scale CMB spectrum is suppressed relative to that in LCDM.

Alleviating the problem of tracker solutions of Galileons

Better compatibility with the Planck CMB data

Observational constraints on GGC

Peirone et al, arXiv:1905.05166



$x_1^{(0)}$, $x_2^{(0)}$, $x_3^{(0)}$ are today's density parameters arising from $-X$, c_2X^2, and $\beta_3\Box\phi/M^3$, respectively.

Not only the χ^2 statistic but also several information criteria (like the Bayesian evidence) support GGC over LCDM (in spite of two additional parameters).

Summary of scalar-tensor dark energy models

The GW170817 event constrained the Hordenski Lagrangin to be

$$L = G_2(\phi, X) + G_3(\phi, X)\Box\phi + G_4(\phi)R$$

- Quintessence and k-essence are not particularly favored over LCDM from the current observational data.
- So far, there were no observational signatures for nonminimally coupled theories (including $f(R)$ gravity).
- The cubic Galileon with the late-time dominance as dark energy is ruled out from observations.
- However, the GGC model is in a better compatibility with the data in comparison to LCDM.

Let's see how future observational data constrain the GGC model further.