Low scale gravity and QCD at high energies

Flying to Hidden Universe

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Gravity and Hierarchy Problem

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In brane models the fundamental quantum gravity scale can be adjusted by varying metric, size, and number of extra-dimensions.

At low energies Standard Model particles must somehow be kept on the brane.
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⋆ We want to see what happens to SM particles in a high energy collision. This is relevant for LHC and high energy air showers.
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- In string models there is not yet a fundamental mechanism to explain compactification.
- Compactification can be due to the breaking of Lorentz and other symmetries, phase transition and formation of defects. Chan et al. 00, West 02
Simplest Brane Model

- We only consider warped models because particles are partially confined.
- Simplest warped brane metric is Randall-Sandrum model:

\[ ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2) \]

\[ R \equiv \frac{1}{\mu}, \quad R' \equiv \frac{e^{\mu L}}{\mu}, \quad z \equiv \frac{1}{\mu} e^{\mu y} \]

- L: Effective bulk size or distance between branes.

- Near branes models with more complex geometries are similar to RS models Palma 06
Remind of some properties of particles spectrum

* Mass spectrum of eigen-modes (Kaluza-Klein modes) for 2-brane models Dubovsky et al. 2000, HZ 02:

\[ |m_n| \approx \mu e^{-\mu L} \left( n\pi + \frac{\pi d}{4} + \frac{3\pi}{4} \right), \quad \Delta m \sim \pi \mu e^{-\mu L} \ll \mu, \; n \gg 1 \]

* Spectrum is roughly continuous.

* Gravitational confinement by warp factor depends on the tension on the brane and spin.

* Vector field (e.g. photons and gluons) can not be localized on the brane by warp factor. Dubovsky & Rubakov 01
Constraining brane model parameters

* Many measurements and observations can constrain brane models:
  - Violation of inverse square law; Smullin, *et al.* 05
  - Electroweak precision measurement; Marandella & Papucci 04
  - Modification of Big Bang Nucleosynthesis and FLRW cosmology; HZ 00, Bratt *et al.* 02 Fairbairn & Goobar 05
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Interaction of ultra high energy cosmic ray has a CM energy \( \sim 10^3 TeV \), and can constrain brane models at energies unavailable to accelerators.
Which scale is probed by UHECRs?

- At high energies nucleon-nucleon collision and Deep Inelastic Scattering (DIS) are dominated by small $x_b \equiv Q^2 / 2p.q$ regime.
- In accelerators the KK-modes can be distinguished only if they have large transverse momentum.
- In an air-shower KK-mode does not need to have a large transverse momentum to be observed.
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* Momentum ordering - smaller fraction of momentum, larger diffraction angle - is the evidence of QCD radiational effect and direct relation between small $x$ and large $x$ physics.

Figure 0: Iancu 06
Difficulties

- At high energies due to high density of partons (gluons) processes are very complicate and mostly non-perturbative, $\alpha_s \ln(1/x_b) > 1$.

- If we can study the evolution of QCD interactions with energy scale, we can relate low energy scale (long distance) observables to interactions at high energy scales (short distance) - presumably TeV scale if incident hadrons are enough energetic.

- Due to non-perturbative properties of QCD, there is no exact formulation of the scale evolution.
Color Glass Condensate (CGC)

- Color Glass Condensate approximation assumes that in light cone coordinates, color charges are concentrated only on a sheet. McLerran & Venugopalan 94

- The origin of color charge is mostly valance quarks, but sea partons integrated up to a scale $\Lambda^+$ also contribute to the total charge. Soft partons (mostly gluons) make a swarm between these sheets.

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This approximation can be applied to any regime and not just what is called saturation as long as we deal with statistical distribution of a large number of partons.
Color Glass Condensate (CGC) in 5-dimension

* QCD is modelled by an effective, classical $SU(3)$ gauge field $A^B$ of gluons. A universal bare coupling in the bulk and on the branes is assumed.

* In Light Cone (LC) gauge:
  \[ A^+ = 0, \quad x^+ = x^0 + x^3, \quad x^- = x^0 - x^3, \quad \vec{x} = \{ x^-, x^\perp, z \} \]

* Classical dynamic equation:
  \[
  [D_A, F^{AB}](x) = \delta^B + \mathcal{W}(x^+, \vec{x})\rho(\vec{x})\mathcal{W}^\dagger_{\Lambda+}(x^+, \vec{x})
  \]
  \[
  \mathcal{W}_{\Lambda+}(x^+, \vec{x}) = T \exp \left\{ ig \int_{x_0^+}^{x^+} d\eta^+ \frac{R^2}{z^2} A^-(\eta^+, \vec{x}) \right\}
  \]

* If fermions are confined to the visible brane, $\rho \neq 0$ only for $z = R'$. But this configuration is inconsistent and is violated by a gauge transformation.
Quantum extension of CGC

* For quantum extension, quantities must be averaged for all possible distribution of charge $\rho(\vec{x})$. Partition function:

$$
Z[j] = \int D\rho P_{\Lambda^+}[\rho] \left\{ \frac{\int^{\Lambda^+} DA\delta(A^+)e^{iS[A,\rho]} - \int d^5x \sqrt{-gA.J}}{\int^{\Lambda^+} DA\delta(A^+)e^{iS[A,\rho]}} \right\}
$$

* When quantum corrections are added, the evolution equation for $P_{\Lambda^+}$ the probability distribution of $\rho(\vec{x})$ can be described as a renormalization group equation: Jalalian Marian et al. 97, Iancu et al. 00

$$
\frac{\delta P_{\tau}[\rho]}{\delta \tau} = \alpha_s \left\{ \frac{1}{2} \frac{\delta^2}{\delta \rho^a_{\tau}(x^\perp,z)\delta \rho^b_{\tau}(x'^\perp,z')} [P_{\tau}\chi^{ab}] - \frac{\delta}{\delta \rho^a_{\tau}(x^\perp,z)} [P_{\tau}\sigma^a] \right\}
$$

$$
\sigma^a = \langle \delta \rho^a \rangle \quad \chi^{ab} = \langle \delta \rho^a \delta \rho^b \rangle \quad \tau \equiv \ln(P^+/\Lambda^+)
$$
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- With this approximation we determine the evolution of gluon distribution in the bulk.
Gluon distribution function in the bulk

* Distribution of gluons in the bulk defined as: Muller 99 HZ 05

\[ x_b G(x_b, Q^2, z) = 2J_1(1) P^+ x_b Q^2 \int d^3 \vec{x} e^{i P^+ x_b x^-} x^{1/2} \int d^3 \vec{x}' \] 
\[ \int_0^L \, dy' e^{\mu y'} \left\langle A^i(x^+, \vec{x}', y') A_i(x^+, \vec{x} - \vec{x}', y - y') \right\rangle \]

* Expectation value of any functional \( O(\rho) \) is

\[ \langle O(\rho) \rangle = N \int D\rho \mathcal{P}[\rho] O(\rho). \]

\[ \left\langle A^i_a(\vec{x}, z) A_{bi}(\vec{x}', z') \right\rangle = \delta_{ab} \chi(\vec{x}, \vec{x}', z, z') \partial_i \partial_{i'} \gamma(x^\perp, x'^\perp, z, z') + \ldots \]

\[ \chi \equiv (zz')^{1/2} \frac{1}{R} \int_{-\infty}^{\max(x^-, x'^-)} dx''^- \sigma_+(x''^-, x'^\perp, z) \sigma_+(x''^-, x'^\perp, z') \]

Brane-World Gravity, Progress and Problems, Portsmouth, Sep. 2006
Gluon distribution function in the bulk

Standard deviation of $\rho$ distribution:

$$T^{ab} \chi_{ab} \approx 4i g^2 T^{ab} F_{ac}^{+i}(x) \langle x | G_{0ij} | y \rangle F_{cb}^{+j}(y) \sim \sigma^2_{\Lambda} \propto \left( \frac{zz'}{R^2} \right)^n$$

$$n \sim \frac{3}{2}$$

$$G_0^{ij-1}(x) = g^{ij} \partial_i B \partial_j B, \quad i = \perp, z$$

$$M_5 = 10^{14} \text{eV}$$

$$M_5 = 10^{15} \text{eV}$$

Fine-tuned RS-model:

$$\log \left( \frac{R'}{R} \right) = \log \left( \frac{M_{pl}}{M_5} \right)$$
Gluon distribution function in the bulk

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\[ \mu = M_5 \]

\[ 2 \times 10^8 \text{eV} \leq |k_\perp| \]

\[ 1.26 \times 10^{10} \text{eV} \]

* Gluon distribution in the bulk normalized to the amplitude of the distribution on the visible brane at \( z = R' \).
In general the probability of $z \neq R'$ is much larger than $z = R'$, and its maximum is toward the brane at $z = R$. From propagator equation:

$$\frac{g_0^2}{g_n^2} \sim \left( \frac{m_n}{\mu} \right)^2 \propto e^{-2\mu L}$$
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* CGC is relevant for non-hadronized gluons. After hadronization coupling between particles changes and propagation will not be the same. This is important for models with macroscopic bulk like fine-tuned RS model, but irrelevant when the bulk size is smaller than \( \Lambda^{-1}_{QCD} \).
Conclusions

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★ If a macroscopic or relatively large higher dimension exits, it should have modified the spectrum of UHECRs and their shower in the terrestrial atmosphere.
★ This put a lower limit of $M_5 \gtrsim 10^3 TeV$ on static RS model.
★ Regarding this limit, should we consider brane models as a solution for hierarchy problem if Electroweak scale is $\sim 1 TeV$?
Back to Hierarchy Problem
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From **Old Hierarchy** we learn that a model should work in its most simplest case, and additional of more complexities must be only limited to its minor aspects.