Brane Black Holes and Holography

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Overview

- Gravity/gauge duality: a quick intro

- Black holes on 3-brane, what do we know?

- The 2d case

- Conclusions

- Things to do
AdS/CFT

(Maldacena, 1997)

Gravity

AdS$_5 \times$ S$_5$ (background)

AdS horizon ("Infinity")

Field Theory

N D3-branes (N large)

Stuck of D3 branes, two descriptions

- Solution of SUGRA
- Field theory generated from the string ending (SU(N) SYM $N=4$)
Adding Gravity

(Witten 1998)

Hawking Flux

Black hole

\[ T \neq 0 \]

SAdS$_5$

Dual description:

- Semiclassical gravity $T \neq 0$
- Thermal CFT (TCFT)

But Hawking flux is not unitary in contrast with TCFT!
Solution

(Maldacena 2003, Hawking 2005)

Partition function

\[ Z = \int Dg D\Phi \ e^{\frac{i}{\hbar} \int d^3 x \left( \frac{\mathcal{L}}{k_m^2} + \mathcal{L}_m(\Phi) \right)} \]

the semiclassical approximation

\[ Z \propto \int D\Phi \ e^{\frac{i}{\hbar} \int d^3 x \mathcal{L}_m(\Phi)} \]

Gravity fixed by \( \delta \mathcal{L}_c = 0 \) (saddle point approx)

However \( \langle \Phi \Phi \rangle \downarrow 0 \), \( t \uparrow \infty \)

\[ \text{GGG shell solutions dominate!} \]
What is a black hole in the CFT?

A BH corresponds to a coarse grained state in the path integral

\[ \downarrow \]

Represent a non unitary theory (c.f. quantum mechanics)

Where is this non unitary theory living?

From the path integral we can construct a non unitary theory by adding UV cut-off

\[ \Rightarrow \text{boundary cut-off on the gravity side} \]
On the UV gravity is localized (Randall and Sundrum 1999)

On the boundary Classical Gravity + CFT

\[ R_{\alpha \beta} - \frac{1}{2} R g_{\alpha \beta} = \langle T_{\alpha \beta} \rangle_{\text{CFT}} \]

(Tanaka 02
Empanon, Fabbri, Kaloper 02)
Randall–Sundrum model (RS) (1999)

\[
\Omega < 0 \quad \Rightarrow \quad 3 \quad \Rightarrow \quad \Omega > 0
\]

\[
\mathcal{L} = \frac{1}{\kappa_{5}^2} \int d^{5}x \sqrt{g}(R - 2\Lambda) + \int_{\Sigma} d^{4}x \sqrt{\bar{g}} \left( \mathcal{L}_m - 2\lambda \right)
\]

\[
R_{\alpha \beta} - \frac{1}{2} g^{\alpha \beta} R = T_{\alpha \beta} + \frac{S_{\alpha \beta}}{\kappa^{2}} + \mathcal{E}_{\alpha \beta} - \Lambda_{4} g_{\alpha \beta}
\]

\[
\mathcal{E}_{\alpha \beta} = W_{\alpha \beta \rho \sigma} \nu^{\rho}_{\mu} \nu^{\sigma}_{\nu}, \quad \Lambda_{4} = \frac{1}{2} (\Lambda + \lambda^{2})
\]

\[
S_{\alpha \beta} = \frac{1}{12} T T_{\alpha \beta} - \frac{1}{4} T_{\gamma} \gamma^{\gamma}_{\beta} + \frac{1}{24} g_{\alpha \beta} \left( 3 T_{\mu} T^{\mu} - T^{2} \right)
\]
Asymptotically Flat D3 brane

Consider a BH on the cutoff

In 4D asymptotically flat BH are not in thermal equilibrium

\[ \Downarrow \]

Impossibility of finding static black holes on the brane (Bruni, Germani, Maartens 01)

An Hawking Flux appears from a collapsing object on the brane (for a very special toy model)

\[ S^2 \text{ is interpreted as } \langle T^4 \rangle \]

(Casalino and Germani 04)
Asymptotically AdS D3-brane

We know that Asymptotically AdS black holes can be in thermal equilibrium

\[
\downarrow
\]

Static BH solutions surrounded by "quantum" matter.

These solutions were indeed obtained (Galfard, Germani, Ishibashi)
(See also Creek, Gregory, Kanti, Nistri)

However, the Quantum interpretation is still unknown.
A special case

$d$-dimensions

\( R_{\alpha \beta} - \frac{1}{2} R g_{\alpha \beta} + \wedge g_{\alpha \beta} = \langle T_{\alpha \beta} \rangle \)

in 2 dimensions

\( R_{\alpha \beta} = \frac{1}{2} R g_{\alpha \beta} \)

\( \wedge g_{\alpha \beta} = \langle T_{\alpha \beta} \rangle_{\text{cft}} \)

The gravity dynamics is purely quantum and a cosmological constant cannot be zero as \( \langle T_{\alpha \beta} \rangle_{\text{cft}} \neq 0 \)

\( \Downarrow \)

No asymptotically flat solution!
Two dimensional quantum BH

(Germani, Procopio '06, hep-th/0605068, PRD)

The theory

\[ A = \frac{1}{2} \int d^2x \sqrt{-g} \left( R - 2\Lambda \right) + \int d^2x \sqrt{-g} L_{\text{CFT}} \]

Gibbons-Hawking term

\[ A_b = - \int d^2x \, a^\alpha b_{\alpha} \left( K + L_b \right) \]

In two dimensions \( L_b \) is a boundary mass.

\[ \delta A_b = 0 \] so we impose

\[ K = -L_b \] in order to make the boundary irrelevant in the path integral.
Semiclassical equations of motion

Semiclassical gravity:

\[ \Lambda g_{\mu\nu} = \langle T_{\mu\nu} \rangle \]

Trace anomaly

\[ \langle T^\mu_\mu \rangle = -\frac{\hbar}{24\pi^2} R \]

Conservation equation

\[ \nabla^\nu \langle T_{\mu\nu} \rangle = 0 \]

gauge freedom

\[ ds^2 = -\delta^2 (u,v) \, du \, dv \]
Explicit Form

\[-\frac{1}{2} \Lambda \Omega^2 = \langle T_{\mu \nu} \rangle\]

\[\sigma = \langle T_{\mu \mu} \rangle = \langle T_{\nu \nu} \rangle\]

\[\langle T_{\mu \mu} \rangle = -\frac{\sigma}{12\pi} \Lambda \delta_{\mu} \delta_{\mu} \Lambda^{-1} + \tilde{\sigma}(m)\]

\[\langle T_{\nu \nu} \rangle = -\frac{\tilde{\sigma}}{12\pi} \Lambda \delta_{\nu} \delta_{\nu} \Lambda^{-1} + \tilde{\nu}(m)\]

\[\hbar^{-1} \langle T^{\alpha \beta} \rangle = -\frac{\Phi}{24\pi} R\]

where

\[\tilde{\sigma} = \langle \hat{T}_{\mu \mu} \rangle \]

\[\tilde{\nu} = \langle \hat{T}_{\nu \nu} \rangle \]

(For a general solution without boundaries see Sanchez 1986)

\[\sigma\] counts the number of fields in the theory + matter, - gravitons
Fix the vacuum state

\[ \tilde{U} = \tilde{V} = \frac{q}{48\pi} \]

where \( q \) is a constant.

One can prove that this corresponds to a constant thermal flux at infinity (Hartle-Hawking state).

Equations can now be solved

\[ \Sigma^2 = 4q \frac{(\delta - \mu) \sqrt{q}}{\lambda^2} \frac{e^{(\delta - \mu) \sqrt{q}}}{(1 + e^{(\delta - \mu) \sqrt{q}})^2} \]

\[ \lambda^2 = \frac{48\pi q}{48\pi} \]

We did not impose the boundary condition yet!
Understanding the physics
Schwarzschild gauge

\[ ds^2 = - \Delta^2 \, d\omega \, d\omega = -g(x) \partial t^2 + \frac{d x^2}{g(x)} \]

set \( q = \lambda^2 N + \Pi^2 \), \( \epsilon = \frac{\sqrt{q}}{2} \)

\[ X_+ = -\sqrt{q} \frac{(1 - e^{(\nu-m)\sqrt{q}})}{x^2} + \frac{\Pi}{x^2} \]

\[ \delta_+ (x_+) = \lambda^2 x_+^2 + 2\pi x_+ - N \]

\[ \frac{\Pi}{x^2} < x_+ < \frac{\sqrt{q}}{x^2} + \frac{\Pi}{x^2} \]

or

\[ X_- = -\sqrt{q} \frac{(1 - e^{(\nu-m)\sqrt{q}})}{x^2} - \frac{\Pi}{x^2} \]

\[ \delta_- (x_-) = \lambda^2 x_-^2 - 2\pi x_- - N \]

\[ -\frac{\Pi}{x^2} < x_- < \frac{\sqrt{q}}{x^2} - \frac{\Pi}{x^2} \]

we can restrict \( -\frac{\Pi}{x^2} < x_- < 0 \)
Boundary

We can now analytically extend

\[-\infty < x^- \leq 0, \quad 0 \leq x^+ < +\infty\]

and join the two patches, we obtain

\[ds^2 = -g(x) \, dt^2 + \frac{dx^2}{g(x)}\]

\[g(x) = \lambda^2 x^2 + 2M|x| - N\]

This is a BH surrounding a boundary mass

\[\mu = \frac{M}{\sqrt{N}} \Rightarrow M_{50}, N_{50}\]
Properties

Horizon: \[ x_h = -\frac{n + \sqrt{n + 4}}{x^2} \]

\[ \Rightarrow \text{Horizon formed by quantum back reactions} \]

If we rescale \[ \tilde{t} = \sqrt{x} t \]

\[ \tilde{x} = \frac{x}{\sqrt{x}} \]

\[ \tilde{g}(\tilde{x}) = x^2 \tilde{x}^2 + 2\mu |\tilde{x}| - 1 \]

\[ -\lambda^2 \text{ is the asymptotical cosmological constant (AdS)} \]

\[ \mu \text{ is the black hole mass.} \]

Maximally extended space

\[ k_x = \int \frac{dx}{\tilde{g}(x)} , \quad m = t - r_x , \quad v = t + r_x \]

\[ U = -\frac{1}{\sqrt{1}} e^{-\sqrt{\mu} u} , \quad V = \frac{1}{\sqrt{\mu}} e^{\sqrt{\mu} v} \]
\[ ds^2 = \frac{-4q}{x^2} \frac{1}{2-\rho \nu} dU dV \]

\[ \text{AolS}_2 \text{ with a boundary in } x=0 \text{ and in thermal equilibrium} \]

Indeed

Calculating Bogolubov transform. From \((\omega, \nu) \rightarrow (U, V)\) one obtains

\[ T = \frac{\hbar \sqrt{q}}{2\pi k_B} \]

\[ C = \frac{d\mu}{dT} = \frac{4\pi^2 k_B^2 T}{\mu + \hbar N} > 0 \]
We found a solution of the full quantum backreaction problem of a BH surrounding a point particle in thermal equilibrium.

\[ \text{AdS/CFT} \]

There must be a i-brane solution of an asymptotically AdS bulk with same properties of the quantum BH.
Set-up

\[ A = \frac{1}{2\kappa_3^2} \int d^3x \left( R + \frac{2}{L^2} \right) \sqrt{-g} - \frac{1}{\kappa_3^2} \int_{\text{brane}} d^3x \left( k + 2\sigma \right) F_{\mu \nu} \]

+ boundary terms on the brane

\[ R_{\mu \nu} - \frac{1}{2} R g_{\mu \nu} = \frac{1}{\ell} g_{\mu \nu} \]

\[ k_{2\rho} = \sigma h_{2\rho} \quad \left( k = \frac{1}{2} \frac{2m}{\ell} g \right) \quad m \perp \text{brane} \]

As we need an horizon in the bulk, the bulk is a BTZ BH

\[ ds^2 = -F(r) dt^2 + \frac{dr^2}{F(r)} + r^2 d\theta^2 \]

\[ F(r) = \frac{r^2}{L^2} - m \quad \theta \equiv \theta + 2\pi \]

\[ 0 < r < +\infty \]

Horizon: \( r_h = \sqrt{mL} \)
One-brane

\[ \Sigma : \Theta - \Phi(r) = 0 \]

\[ m_\pm = \pm A (0, -\Phi^1, 1) \]

\[ A = r (4^{1/4} r^2 F(r)+1)^{-1/2} \]

\[ K_{\rho \phi} = 0 \quad \text{hcp} \]

\[ \Phi_{\pm}(r) = \pm \frac{1}{\sqrt{m}} \ln \left( \frac{2 + \sqrt{m_2 + 1}}{r} \right) \]

\[ \alpha = 2 \sigma^2 L^3 m \quad \beta = 4 \sigma^2 L^2 m (1-\sigma^2 L^2) \]

Moreover \( \Phi_{\pm}(r) \equiv \Phi_{\pm}(r) + 2\pi \)

The induced space time is asymptotically AdS_2

\[ \Lambda_2 = -\frac{1}{\sigma^2} (1-\sigma^2 L^2) \]
Understanding the brane

cartesian coords

The brane goes around n times

We define the boundary

cusp generating a point particle

P < ∞
We want \( P < \infty \)

\[
\frac{1}{2\pi \Gamma_m} \ln \left( 4 \frac{L^2}{\sigma^2} m (1 - \sigma^2 L^2) \right) \neq \text{Integer} \leftarrow \infty
\]

\( \Lambda_2 \neq 0! \) like in the quantum case.

The induced metric

\[
 ds^2 = - \left( x^2 + 2 \sqrt{1x1 - N} \right) dt^2 + \frac{dx^2}{x^2 + 2 \sqrt{1x1 - N}}
\]

where \( x^2 = \frac{1 - \sigma^2 L^2}{L^2} \),

\[
\Pi = \frac{1}{L} \sqrt{\frac{p^2}{L^2} \left( 1 - \sigma^2 L^2 \right) + \sigma^2 L^2 m}
\]

\[
N = m - \frac{p^2}{L^2}
\]
Black hole

Imposing now the positivity of the boundary mass
\[ \mu = \frac{M}{N^{1/2}} \]
we find that
\[ P < R_{\text{h}}^{\text{BTZ}} \]
therefore the brane BH and the bulk one share the same horizon!

This BH is equivalent to the quantum one!
Moreover to test more the duality we expect
\[ T^{\text{BTZ}} = T^{\text{brane}} \]

Substituting \( X^2, N, T \) with the relations found before we indeed find
Info about the boundary theory

If we now believe in the duality we can define:

2d AdS/CFT, central charge

\[ \gamma = \frac{12\pi L}{\kappa^2} \geq 0 \]

The boundary theory is dominated by matter fields.

Equating the cosmological constant:

\[ \Lambda = \kappa \frac{1 - \sigma^2 L^2}{\sigma L} \geq 0 \]

\( \Lambda \geq 0 \) and \( \sigma \geq 0 \) cannot be obtained classically with our BH solution.

\[ \downarrow \]

Our BH is purely quantum!
Conclusions

- RS scenario is a good framework to understand semiclassical gravity via holography.

- In 4D it seems that no asymptotically flat BH might exist.

- In 2D one can solve the full quantum back reaction problem of a quantum BH in thermal equilibrium and compare with a 1-brane on a BTZ background.

- Info on the boundary theory.
To do

1. What are the holographic dual of the asymptotically AdS brane solution?

2. In our 2D model the BH is non-trivial because a boundary mass.
   Can we study the evaporation of this BH both from the classical and quantum case? (Work in progress Germani, Procopio)

↓

It seems that the BH stop its evaporation before disappear ⇒ No info lost!