On the thermal boundary condition of the wave function of the Universe and the string landscape (work in progress)

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Introduction

FRW radiation-filled Universes

Brustein and de Alwis thermal boundary condition

A Broader Analysis of BA boundary condition

DeWitt’s argument and the “tunnelling wave function”

Including moduli fields

Conclusions

Outline

1. Introduction
2. FRW radiation-filled Universes
3. Brustein and de Alwis thermal boundary condition
4. A Broader Analysis of BA boundary condition
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6. Including moduli fields
7. Conclusions
Introduction and motivation

- The landscape “problem”: existence of a multiverse of vast solutions to the string theory
- How to select a Universe or a class from the multiverse?
- BA proposed a dynamical principle based on the thermal boundary condition of the wave function of the Universe
- The thermal boundary condition has been applied to a very particular physical case
- What happens in the more general case?
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A classical FRW radiation-filled Universe

- Closed FRW Universe filled with radiation
  \[ \rho = \frac{3\tilde{K}}{(8\pi G a^4)} \]

- There are two turning points for \( 0 < 4\tilde{K}\lambda < 1 \)
  \( (V_0 = a^2 - \lambda a^4) \)
  \[ a_{\pm} = \frac{1 \pm m}{2\lambda}, \quad m = \sqrt{1 - 4\tilde{K}\lambda} \]

- There are two Lorentzian solutions: a collapsing FRW Universe and an asymptotically de Sitter Universe

Two classically allowed Universes separated by a potential barrier are classically disconnected. However, this does not mean they cannot be connected quantum mechanically (Rubakov 84; Halliwell and Laflamme 89; MBL, Garay and González-Díaz 02)
Transition amplitude for FRW Universes with radiation and a cosmological constant-1-

- The transition amplitude, \( \mathcal{A} \), depends on the boundary condition (c.f. Vilenkin 98; MBL and Moniz 04)
- Within a WKB approximation

\[
\mathcal{A} = \exp \left[ \epsilon \frac{\pi}{\sqrt{2G}} \tilde{K} f(m(\tilde{K}\lambda)) \right]
\]

\( \epsilon = 1, -1 \) for HH and tunnelling wave functions respectively

- For a given amount of radiation \( \tilde{K} \) (notice \( f \) is decreasing): i) HH wave function suggests a vanishing \( \lambda \) and ii) tunnelling wave function suggests a large \( \lambda \) \( (\lambda \sim 1/(4\tilde{K})) \)
Transition amplitude for FRW Universes with radiation and a cosmological constant-2-

- More on the transition amplitude
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On the thermal boundary condition of the wave function of the Universe

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BA proposal-1-

- The Universe emerges from the string era in a thermal state above the HH vacuum
- Self-consistency $\implies$ the highest energy density on the region of interest $(a_- < a < a_+)$ is below the string scale $(\rho(a_-) < M^4_s)$
- BA supposed $\rho(a_-) \propto n_{dof}$
- BA approximated $a_-^2 \sim a_+^2 \sim 1/(2\lambda)$

A large amount of radiation $4\tilde{K}\lambda \sim 1$

- In conclusion $\rho = \frac{3\tilde{K}}{8\pi G} \frac{1}{a^4}$, where $\tilde{K} \sim \frac{\nu}{\lambda^2}$ and $\nu \equiv n_{dof} b^{-4} \pi G M^4_s$ $\implies$

the radiation term depends on the cosmological constant

(Blustein and de Alwis 05)
BA proposal-2-

- The transition amplitude for the thermal boundary condition
  \[ \mathcal{A} = \exp \left[ \frac{\pi}{\sqrt{2} \nu G} g\left( m \left( \frac{\nu}{\lambda} \right) \right) \right] \]

- The thermal boundary condition prefers a non-vanishing \( \lambda \sim 8.33 \nu \)

- BA proposal switches the roles of HH and tunnelling wave functions

- A different question has been addressed: What is the most likely \( \lambda \) for a given \( \nu \)?

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On the thermal boundary condition of the wave function of the Universe
The Universe emerges from the string era in a thermal state above HH or tunnelling vacuum

Self-consistency $\implies$ the highest energy density on the region of interest ($a^- < a < a^+$) is below the string scale ($\rho(a^-) < M_s^4$)

Suppose $\rho(a^-) \propto n_{dof}$

No restriction on the amount of radiation $\tilde{K}$ ($0 < 4\tilde{K}\lambda \leq 1$)
The radiation energy density

\[ \rho = \frac{3\tilde{K}}{8\pi G \ a^4} \]

\[ \tilde{K} = \frac{4\nu \lambda^{-2}}{(1 + 4\nu \lambda^{-1})^2} \]

- Radiation term depends on the cosmological constant
- No restriction on the amount of radiation
Generalised thermal boundary condition-3-: HH wave function

- The transition amplitude for the thermal boundary condition
  \[ \mathcal{A} = \exp \left[ + \frac{\pi}{\sqrt{2} \nu G} g(m(\frac{\nu}{\lambda})) \right] \]

- The generalised thermal boundary condition applied to HH wave function favours a vanishing cosmological constant \((\nu/\lambda \gg 1 \text{ and } \tilde{K} \sim 1/(4\nu))\)

- The generalised thermal boundary condition recovers the expected result
  \[ \mathcal{A} \sim \exp \left\{ \frac{\pi}{G\lambda} \left[ 1 - \frac{3}{16} \left( 6 \ln 2 - \ln \frac{1}{\nu} + 1 \right) \frac{1}{\nu} \right] \right\} \]
Generalised thermal boundary condition-4-: Tunnelling wave function

The transition amplitude for the thermal boundary condition

\[ A = \exp \left[ -\frac{\pi}{\sqrt{2} \nu G} g\left( m\left( \frac{\nu}{\lambda} \right) \right) \right] \]

The generalised thermal boundary condition applied to tunnelling wave function favours (i) \( 4\nu/\lambda \rightarrow 1 \) (\( 4\tilde{K}\lambda \rightarrow 1 \)): there is no tunnelling (both turning points coincide)

The generalised thermal boundary condition recovers the expected result

\[ A \sim \exp \left[ -\frac{3\pi^2}{32\sqrt{2}} \frac{1}{G\lambda} \left( 1 - 4\frac{\nu}{\lambda} \right)^2 \right] \]
The generalised thermal boundary condition applied to the tunnelling wave function favours also (ii) a large cosmological constant $\nu/\lambda \ll 1$ and a small amount of radiation as measured by $\tilde{K}$ ($4\tilde{K}\lambda \ll 1$)

$$A \sim \exp \left\{ -\frac{\pi}{G\lambda} \left[ 1 - 3 \left( 2 \ln 2 - \ln \frac{\nu}{\lambda} + 1 \right) \frac{\nu}{\lambda} \right] \right\}$$

Which of the two possibilities is more likely?
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DeWitt’s argument and the “tunnelling wave function”-1-

- DeWitt’s argument: A divergence in a classically allowed region is neutralised by making it quantum mechanically inaccessible (c.f. Davidson et al 99)
- The Ricci curvature diverges at small scale factors
- DeWitt’s argument applied to the tunnelling wave function (once the generalised thermal boundary condition is imposed) \[ V(a) = a^2 - \lambda a^4 - \tilde{K} \]

\[ 1 \ll \exp \left[ \frac{3\pi}{2G} \int_{a_-}^{a_+} \sqrt{V(a)} \, da \right] \]

\[ \frac{3\pi}{2G} \int_{0}^{a_-} \sqrt{-V(a)} \, da = \frac{3\pi}{4} + \tilde{n}\pi, \quad \tilde{n} \in \mathbb{N} \]
Then

- No tunnelling \((4\nu/\lambda \to 1)\) is incompatible
- Large cosmological constant \((\nu/\lambda \ll 1)\) and a small amount of radiation as measured by \(\tilde{K} (\tilde{K}\lambda \ll 1)\) is compatible if

\[
1 \ll \exp \left( \frac{\pi M_P^2}{2\lambda} \right)
\]
Including moduli fields-1-

- We expect to have a moduli dependent potential $\lambda(\phi)$ rather than a cosmological constant.
- $M_p/M_s$ will also depend on $\phi$.
  - Is it possible to constrain (at least qualitatively) the behaviour of the Universe after the tunnelling with the moduli field being considered together with imposing the generalised thermal boundary condition?
  - Is it possible the emergence of an accelerating Universe?
The moduli potential is flat enough

\[ \left| \frac{d \lambda}{d \phi} \right| \ll |\lambda(\phi)| \]

Transition amplitude \( \lambda \rightarrow \lambda(\phi) \) (Vilenkin 88)

Tunnelling wave function \( (\sigma^2 = (2G)/(3\pi)) \)

\[ \frac{2\pi}{3} \frac{n_{dof}}{b^4} \left( \frac{M_S}{M_p} \right)^4 (\phi) \ll \frac{3\pi}{2} \sigma^2 \lambda(\phi) \]

HH wave function

\[ \frac{2\pi}{3} \frac{n_{dof}}{b^4} \left( \frac{M_S}{M_p} \right)^4 (\phi) \gg \frac{3\pi}{2} \sigma^2 \lambda(\phi) \]
The moduli potentials are in general quite steep leading to a dominance of the kinetic energy over the potential \( \Rightarrow \) **No emergence of an accelerating Universe**

However, the moduli potentials are flat close to a **positive** extremum of the potential

The generalised thermal boundary condition, when applied to the wave function of the Universe, produces less restrictive conditions than the one of BA.
Including moduli fields-4-: Application to KKLT-like model

The effective moduli potential (KKLT 03)

\[ \tilde{V} = \frac{a Ce^{-a T_r}}{2 T_r^2} \left[ W_0 + \left( \frac{1}{3} T_r a + 1 \right) Ce^{-a T_r} \right] + \frac{d}{T_r^3} \]

Can the Universe tunnel towards the tails of the potential?

- No: The potential is too steep
- The answer is independent of the HH or tunnelling wave functions.
can the tunnelling end points correspond to an extremum of KKLT potential (dS minimum very small)?

Tunnelling wave function:

\[ \frac{4}{9} \frac{n_{\text{dof}}}{b^4} \ll d + \frac{aCT_re^{-aTr}}{2} \left[ W_0 + \left( \frac{1}{3} T_r a + 1 \right) Ce^{-aTr} \right] \]

is not fulfilled at neither extremum

HH wave function:

\[ \frac{4}{9} \frac{n_{\text{dof}}}{b^4} \gg d + \frac{aCT_re^{-aTr}}{2} \left[ W_0 + \left( \frac{1}{3} T_r a + 1 \right) Ce^{-aTr} \right] \]

is fulfilled at both extremum
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Conclusions and summary-1-

- We extend the thermal boundary condition of the wave function of the Universe.
- Unlike the “restricted” thermal boundary condition that predicts a nonvanishing cosmological constant for the HH wave function, the generalised thermal boundary condition of the wave function of the Universe predicts the usual result.
- Similar result for the generalised thermal boundary condition applied to the tunnelling wave function + the possibility of the tunnelling wave function to favour a Universe with a large cosmological constant and a small amount of radiation (preferred once the DeWitt’s argument is applied)
Conclusions and summary-2-

- We have applied the generalised thermal boundary condition to more realistic models where rather than a cosmological constant we have a moduli dependent potential.

- The generalised thermal boundary condition, when applied to the wave function of the Universe, produces less restrictive conditions than the one proposed by BA.

- Application to KKLT model:
  - The Universe cannot tunnels towards the tail of the potential because of its steepness (independent of HH or tunnelling wave function)
  - The tunnelling end points cannot correspond to the extremum of the potential if we choose the tunnelling wave function
  - The tunnelling end points can correspond to the extremum of the potential if we choose the HH wave function